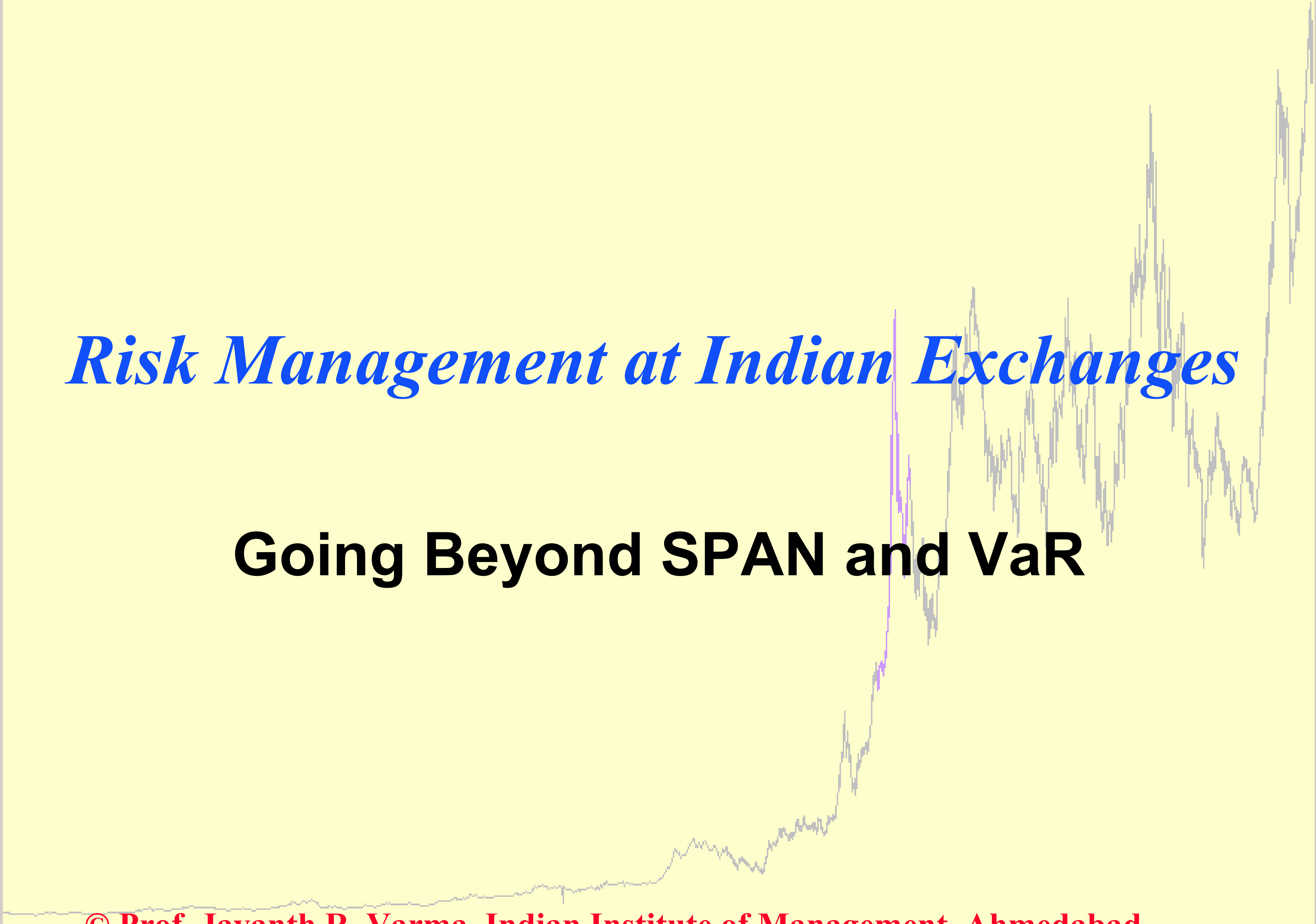
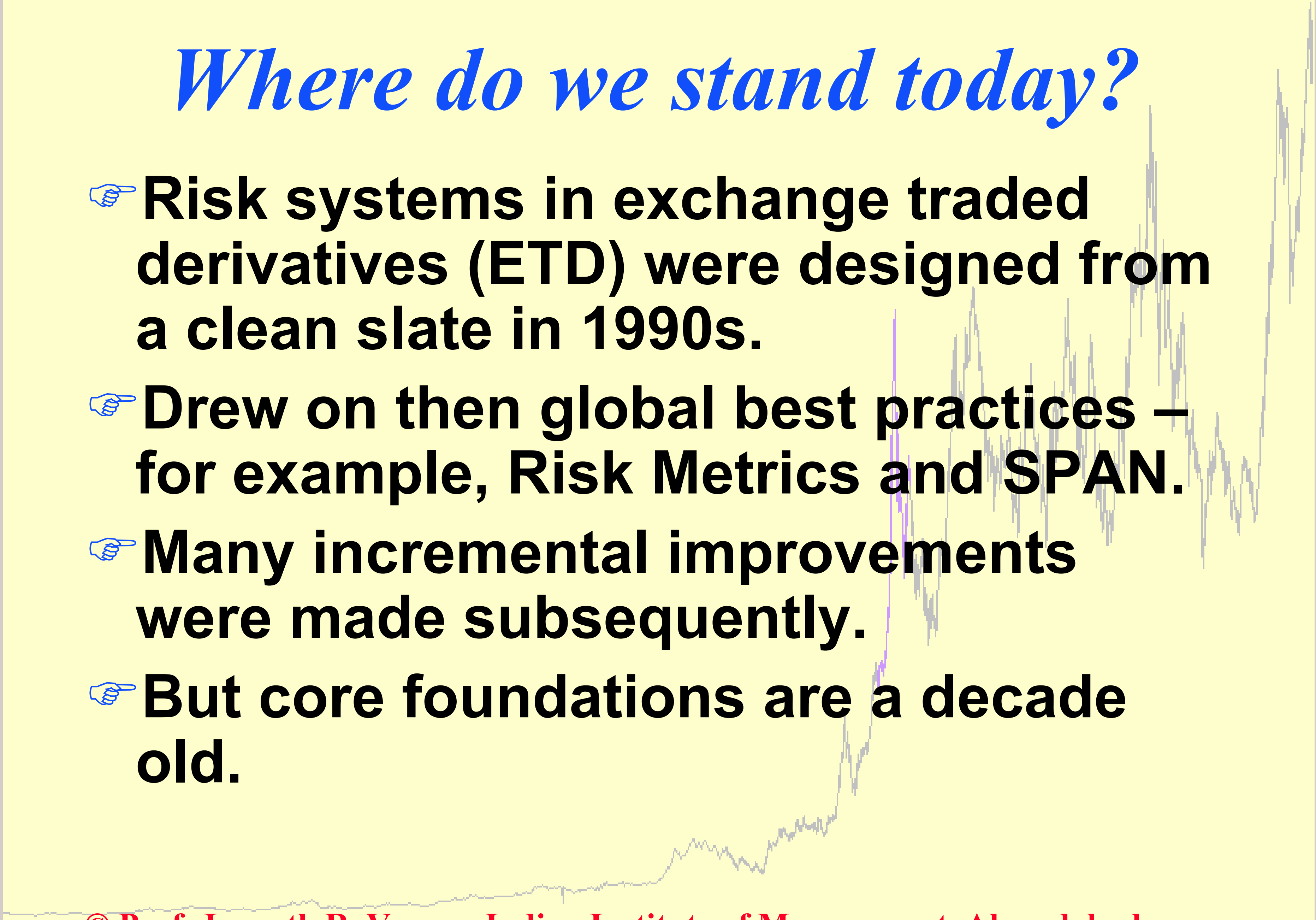


Risk Management at Indian Exchanges

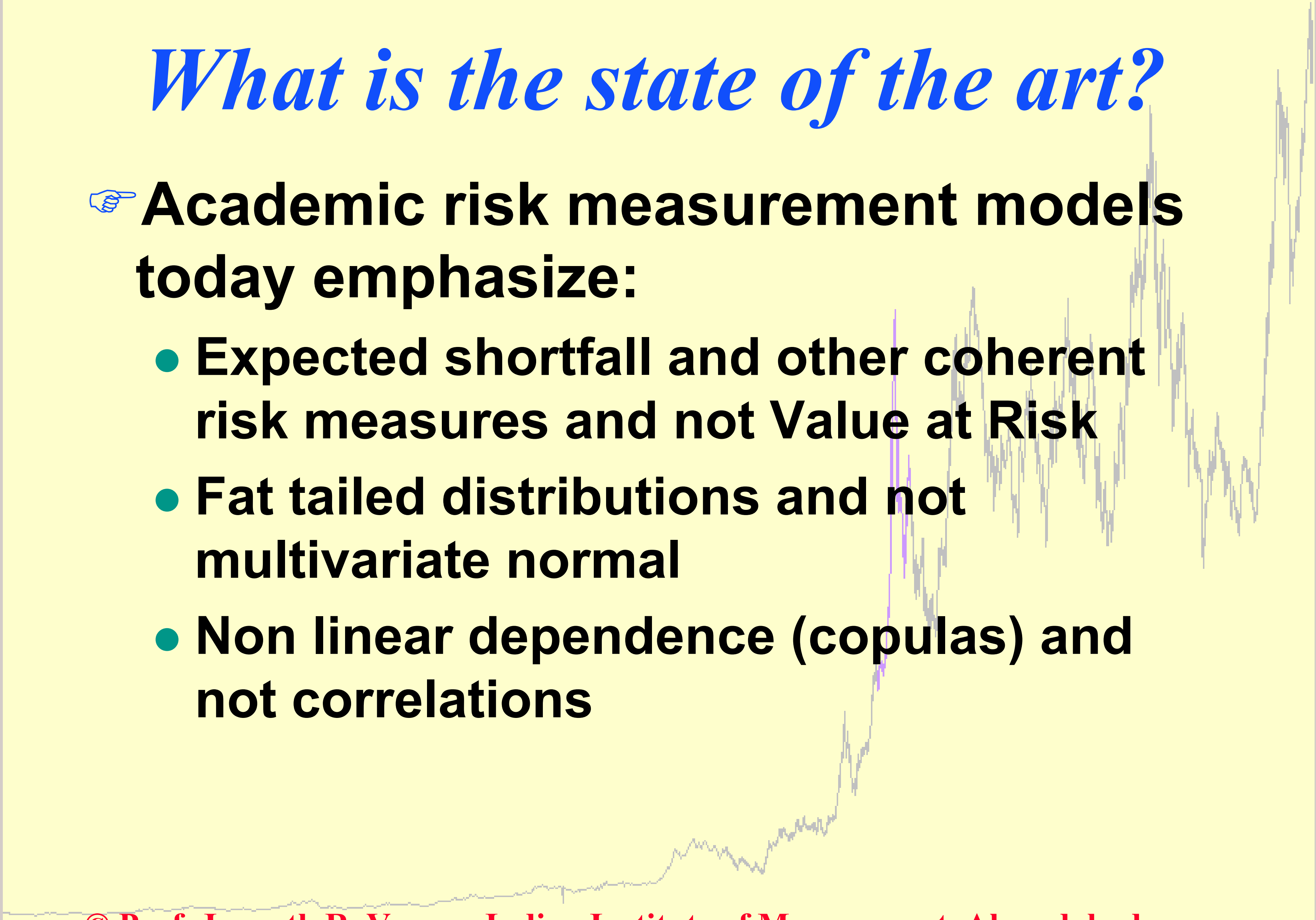
Going Beyond SPAN and VaR

Where do we stand today?



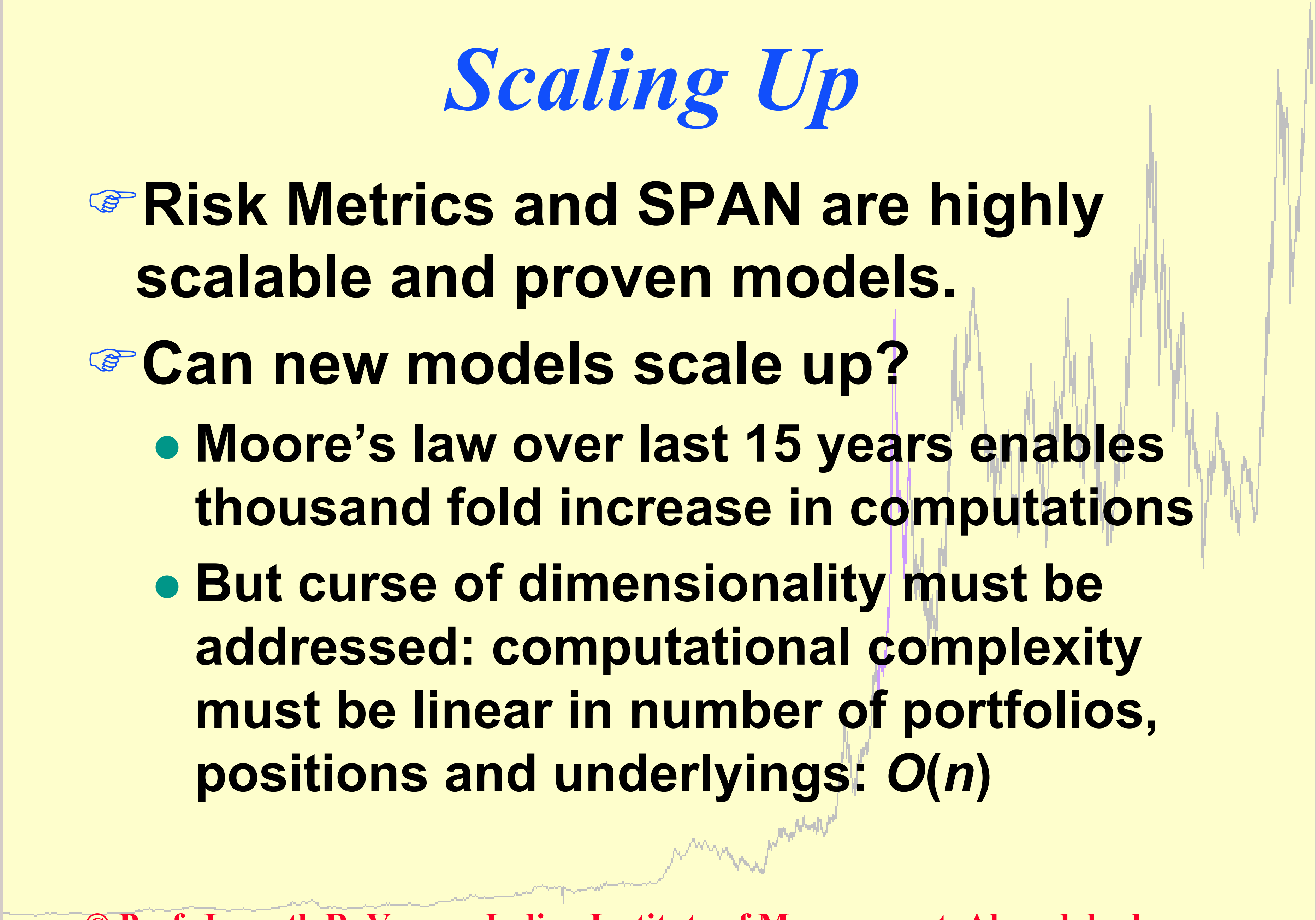
- **Risk systems in exchange traded derivatives (ETD) were designed from a clean slate in 1990s.**
- **Drew on then global best practices – for example, Risk Metrics and SPAN.**
- **Many incremental improvements were made subsequently.**
- **But core foundations are a decade old.**

What is the state of the art?



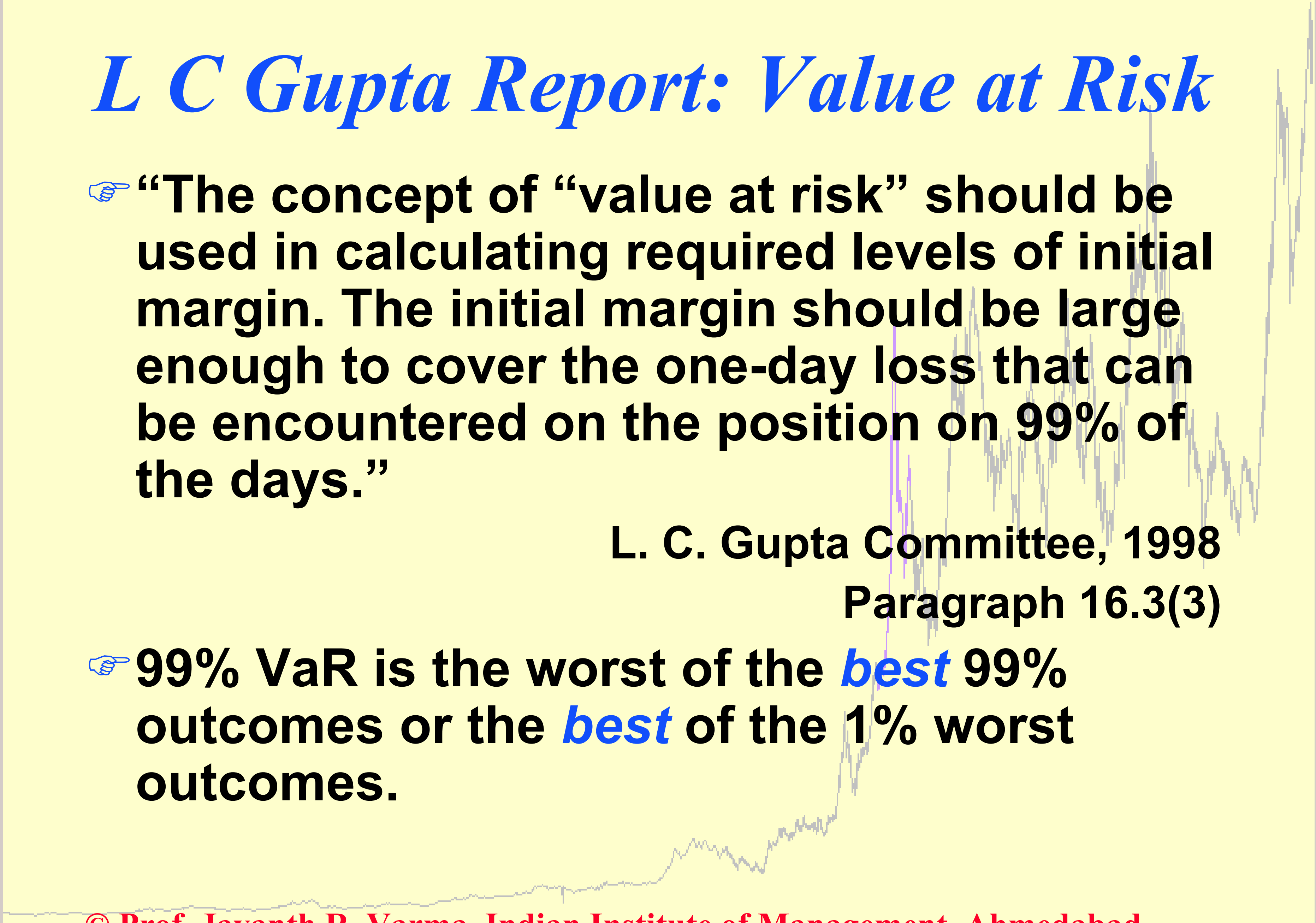
- **Academic risk measurement models today emphasize:**
  - **Expected shortfall and other coherent risk measures and not Value at Risk**
  - **Fat tailed distributions and not multivariate normal**
  - **Non linear dependence (copulas) and not correlations**

Scaling Up



- **Risk Metrics and SPAN are highly scalable and proven models.**
- **Can new models scale up?**
  - **Moore’s law over last 15 years enables thousand fold increase in computations**
  - **But curse of dimensionality must be addressed: computational complexity must be linear in number of portfolios, positions and underlyings: $O(n)$**

L C Gupta Report: Value at Risk

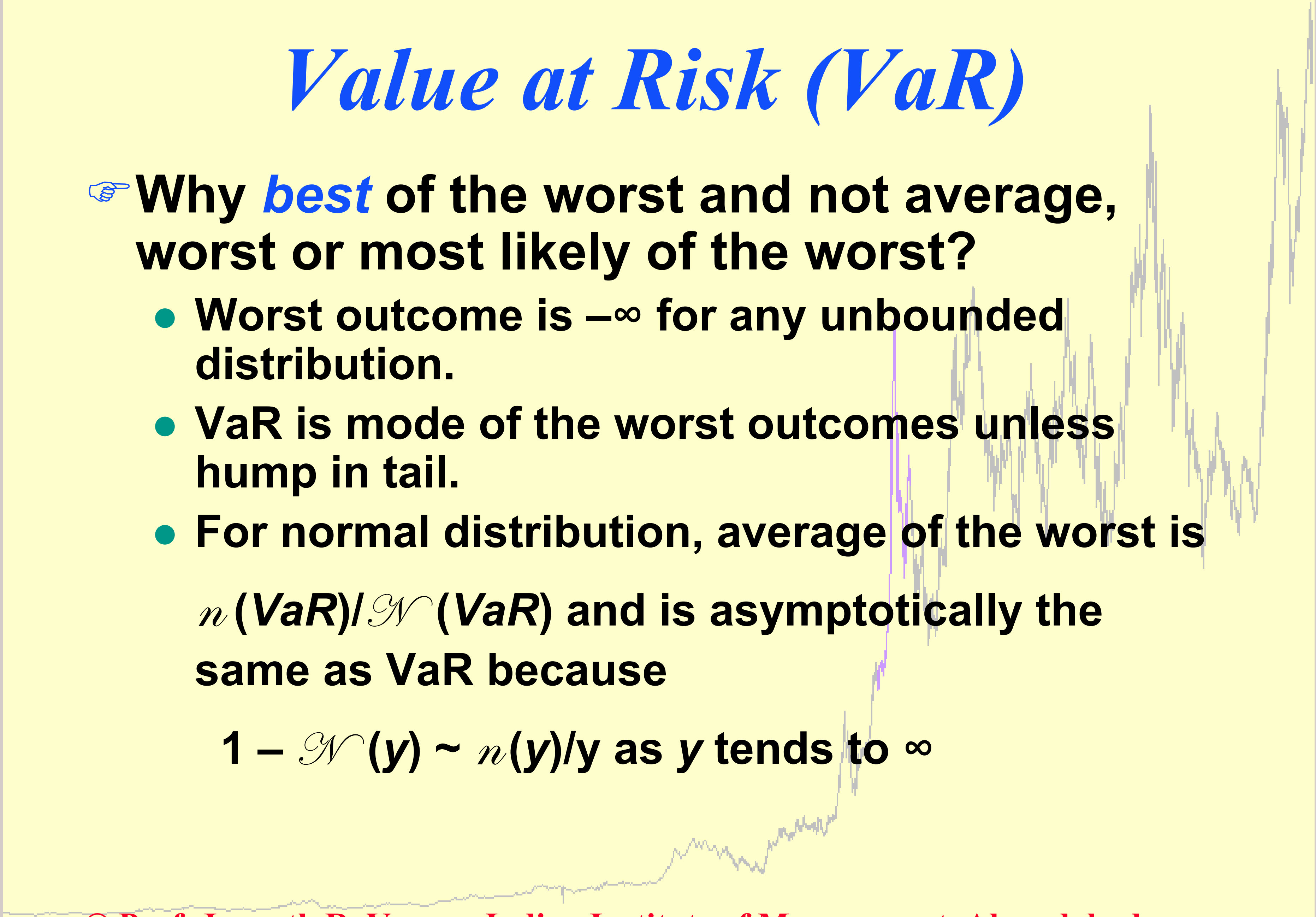


- "The concept of "value at risk" should be used in calculating required levels of initial margin. The initial margin should be large enough to cover the one-day loss that can be encountered on the position on 99% of the days."

  L. C. Gupta Committee, 1998
  Paragraph 16.3(3)

- 99% VaR is the worst of the *best* 99% outcomes or the *best* of the 1% worst outcomes.

Value at Risk (VaR)



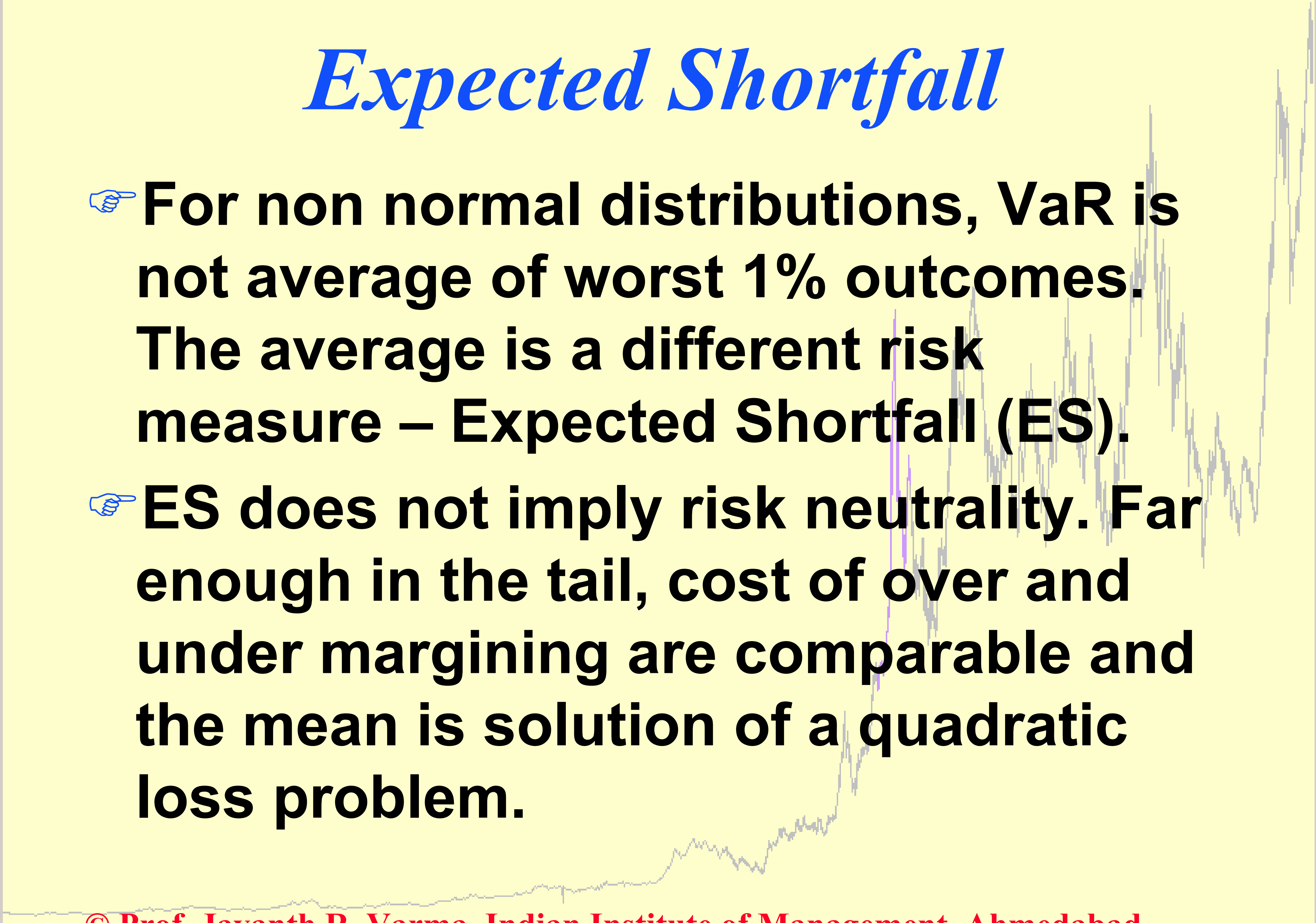
☞ Why *best* of the worst and not average, worst or most likely of the worst?

- Worst outcome is –∞ for any unbounded distribution.
- VaR is mode of the worst outcomes unless hump in tail.
- For normal distribution, average of the worst is

  $n(VaR)/\mathcal{N}(VaR)$ and is asymptotically the same as VaR because

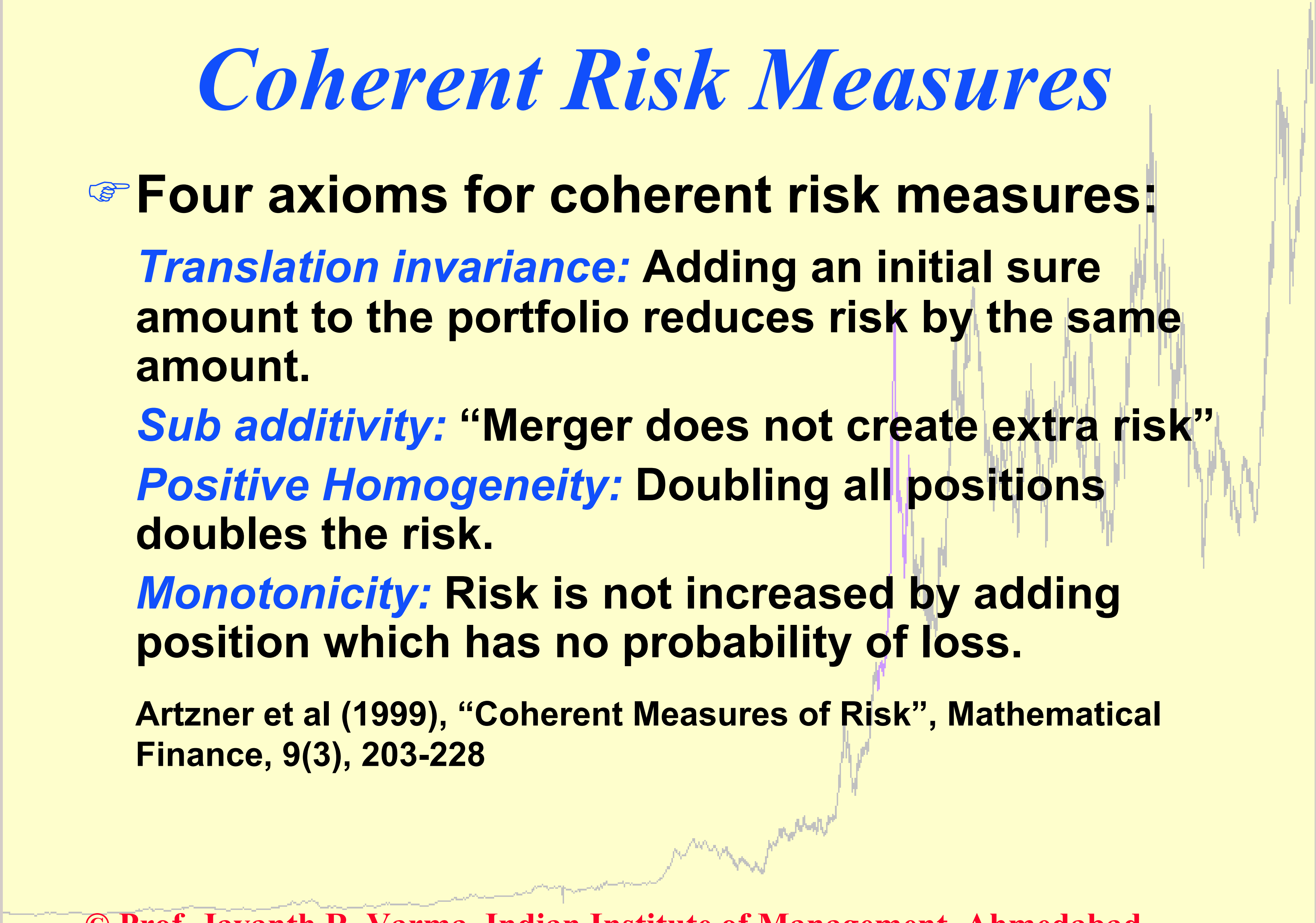
  $1 - \mathcal{N}(y) \sim n(y)/y$ as y tends to ∞

Expected Shortfall



- For non normal distributions, VaR is not average of worst 1% outcomes. The average is a different risk measure – Expected Shortfall (ES).
- ES does not imply risk neutrality. Far enough in the tail, cost of over and under margining are comparable and the mean is solution of a quadratic loss problem.

Coherent Risk Measures



- Four axioms for coherent risk measures:

  Translation invariance: **Adding an initial sure amount to the portfolio reduces risk by the same amount.**

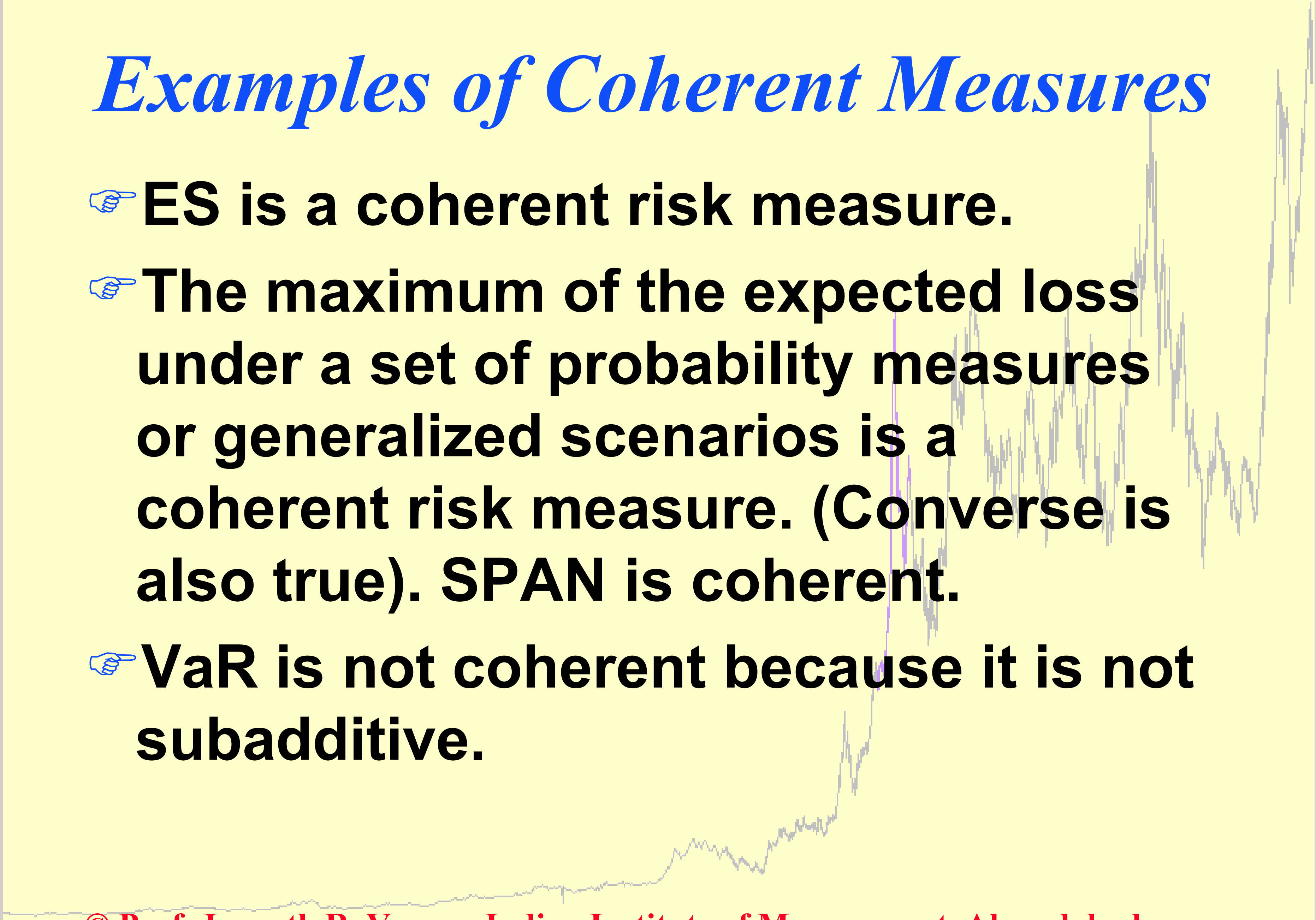
  Sub additivity: **"Merger does not create extra risk"**

  Positive Homogeneity: **Doubling all positions doubles the risk.**

  Monotonicity: **Risk is not increased by adding position which has no probability of loss.**

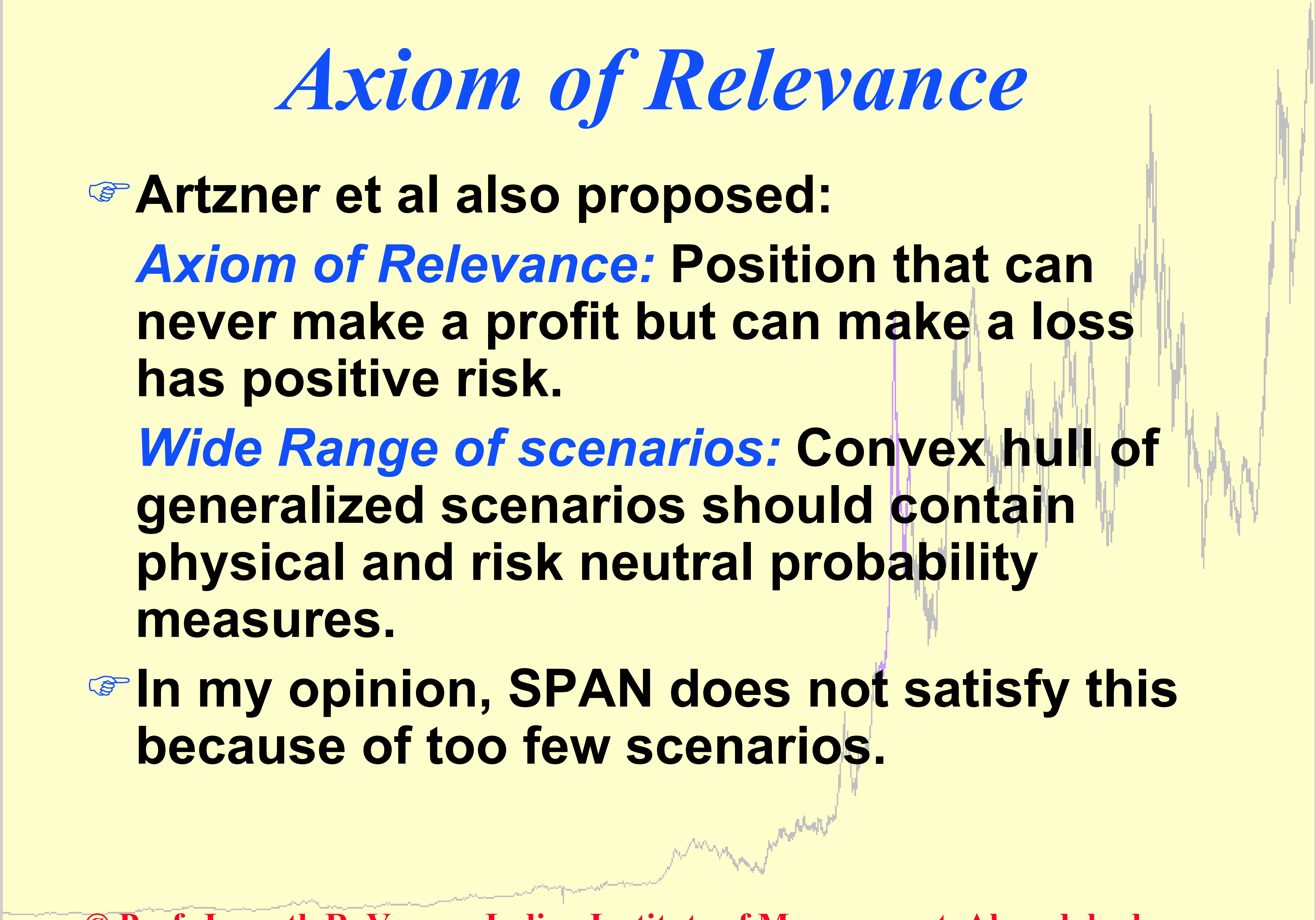
  Artzner et al (1999), "Coherent Measures of Risk", Mathematical Finance, 9(3), 203-228

Examples of Coherent Measures



- ES is a coherent risk measure.
- The maximum of the expected loss under a set of probability measures or generalized scenarios is a coherent risk measure. (Converse is also true). SPAN is coherent.
- VaR is not coherent because it is not subadditive.

Axiom of Relevance



- **Artzner et al also proposed:**

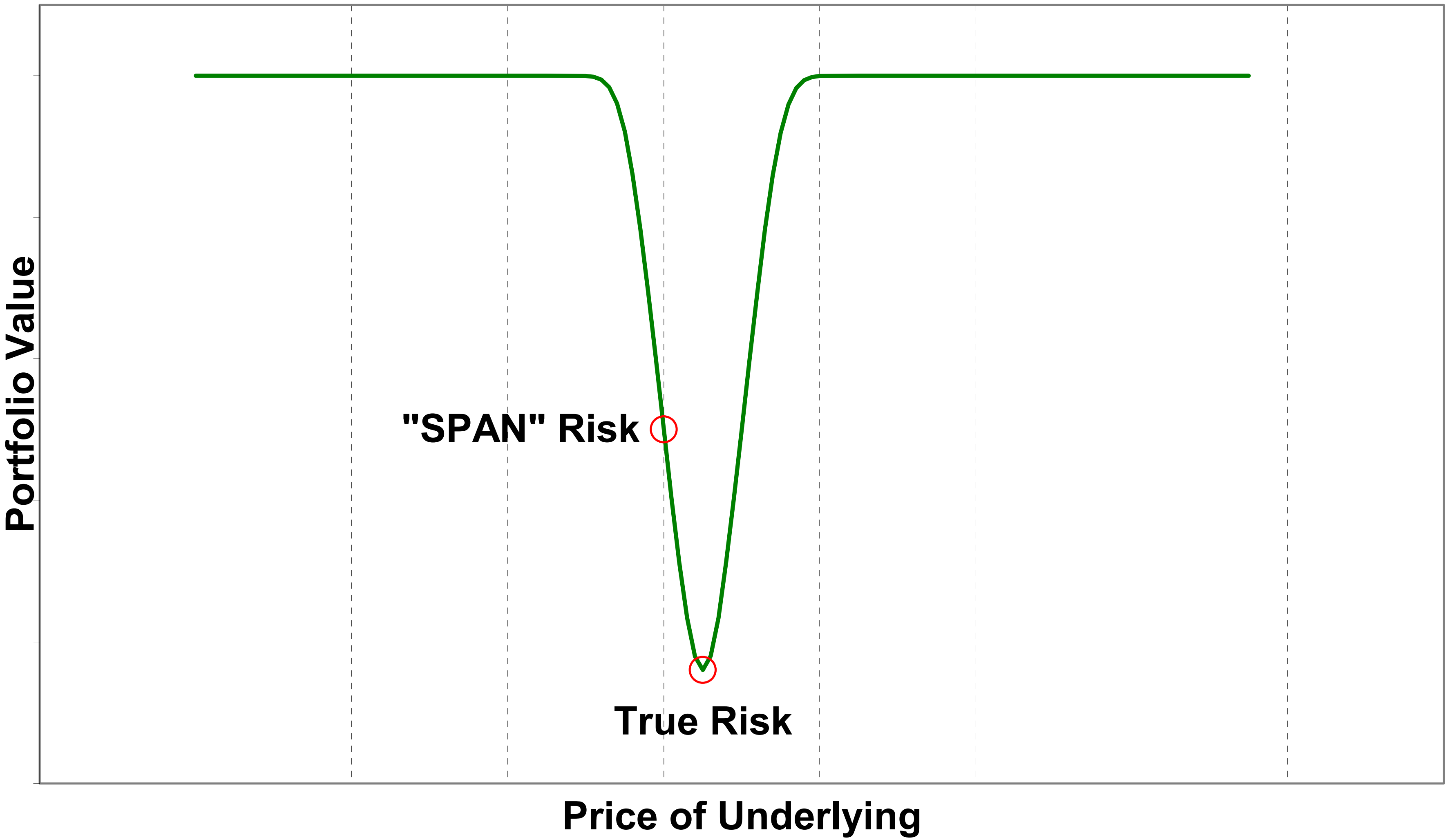
  Axiom of Relevance: **Position that can never make a profit but can make a loss has positive risk.**

  Wide Range of scenarios: **Convex hull of generalized scenarios should contain physical and risk neutral probability measures.**

- **In my opinion, SPAN does not satisfy this because of too few scenarios.**

SPAN Underestimates the Risk of a Short Butterfly

Dotted lines are SPAN price scenarios

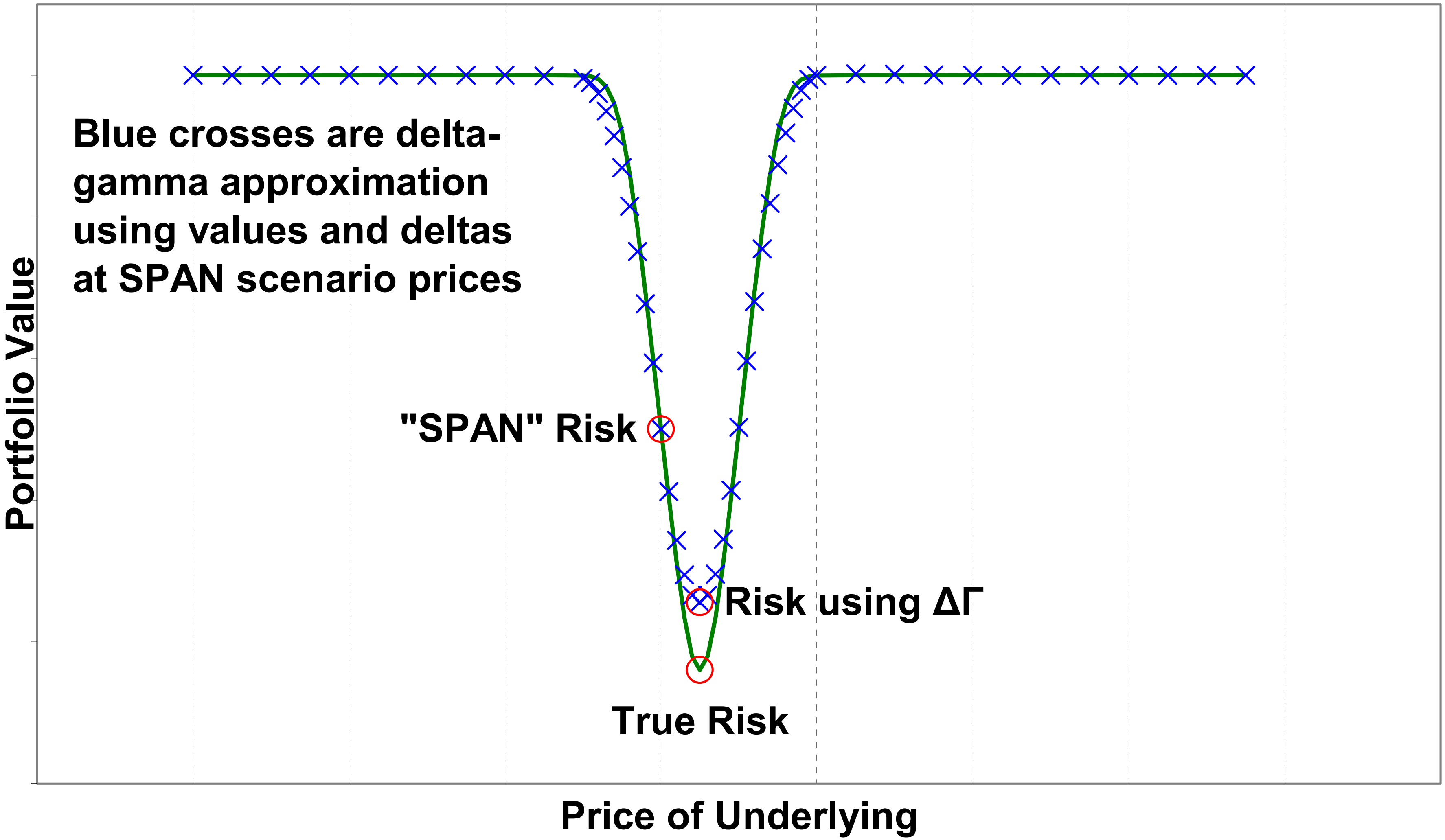


Too Few Scenarios in SPAN

- If price scanning range is set at ±3σ, then there are no scenarios between 0 and σ which covers a probability of 34%.
- Possible Solutions:
  - Increase number of scenarios (say at each percentile)
  - Use a delta-gamma approximation
- Probably, we should do both.

Improved Estimate of the Risk of a Short Butterfly

Dotted lines are SPAN price scenarios

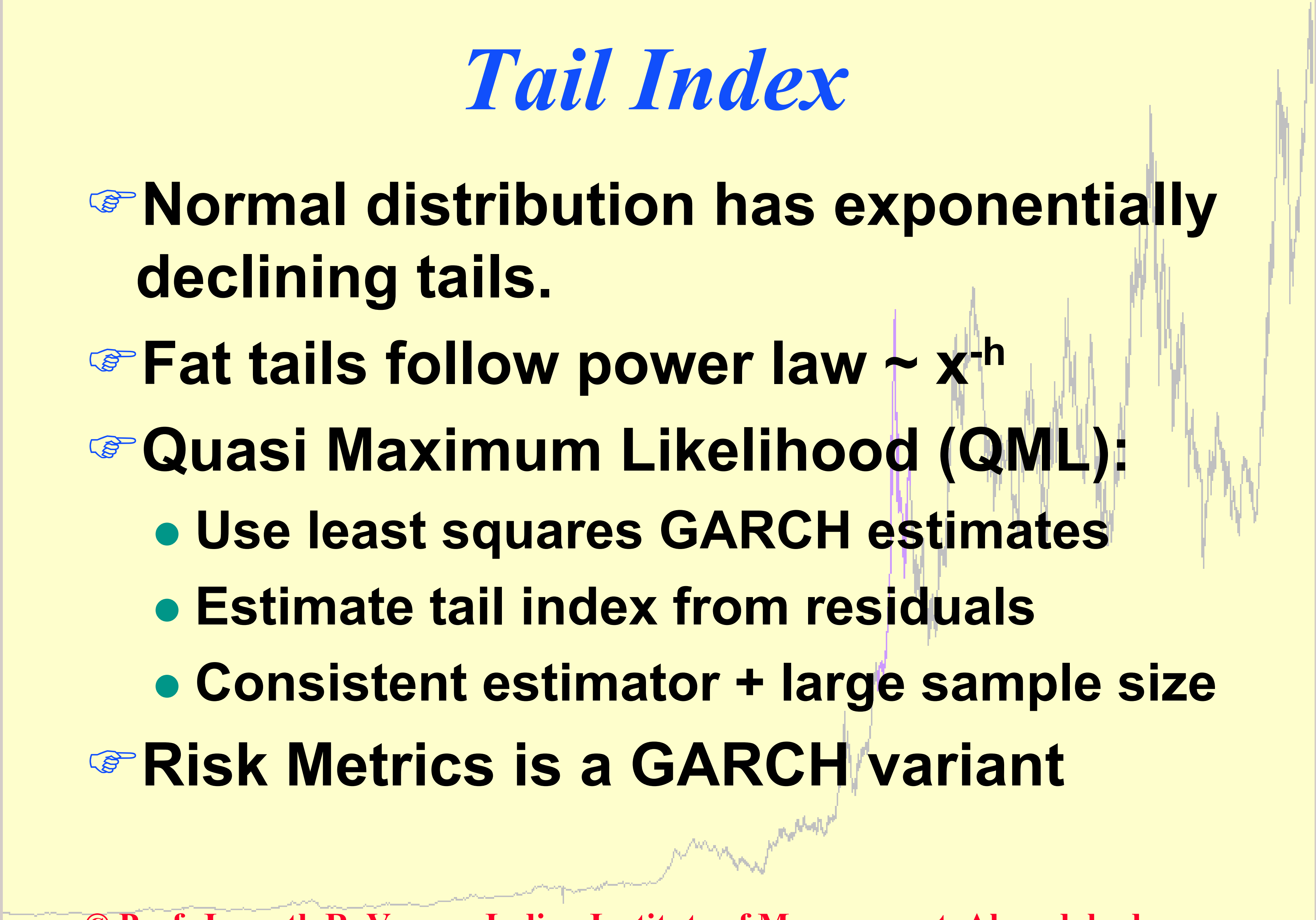


Blue crosses are delta-gamma approximation using values and deltas at SPAN scenario prices

From VaR to SPAN to ES

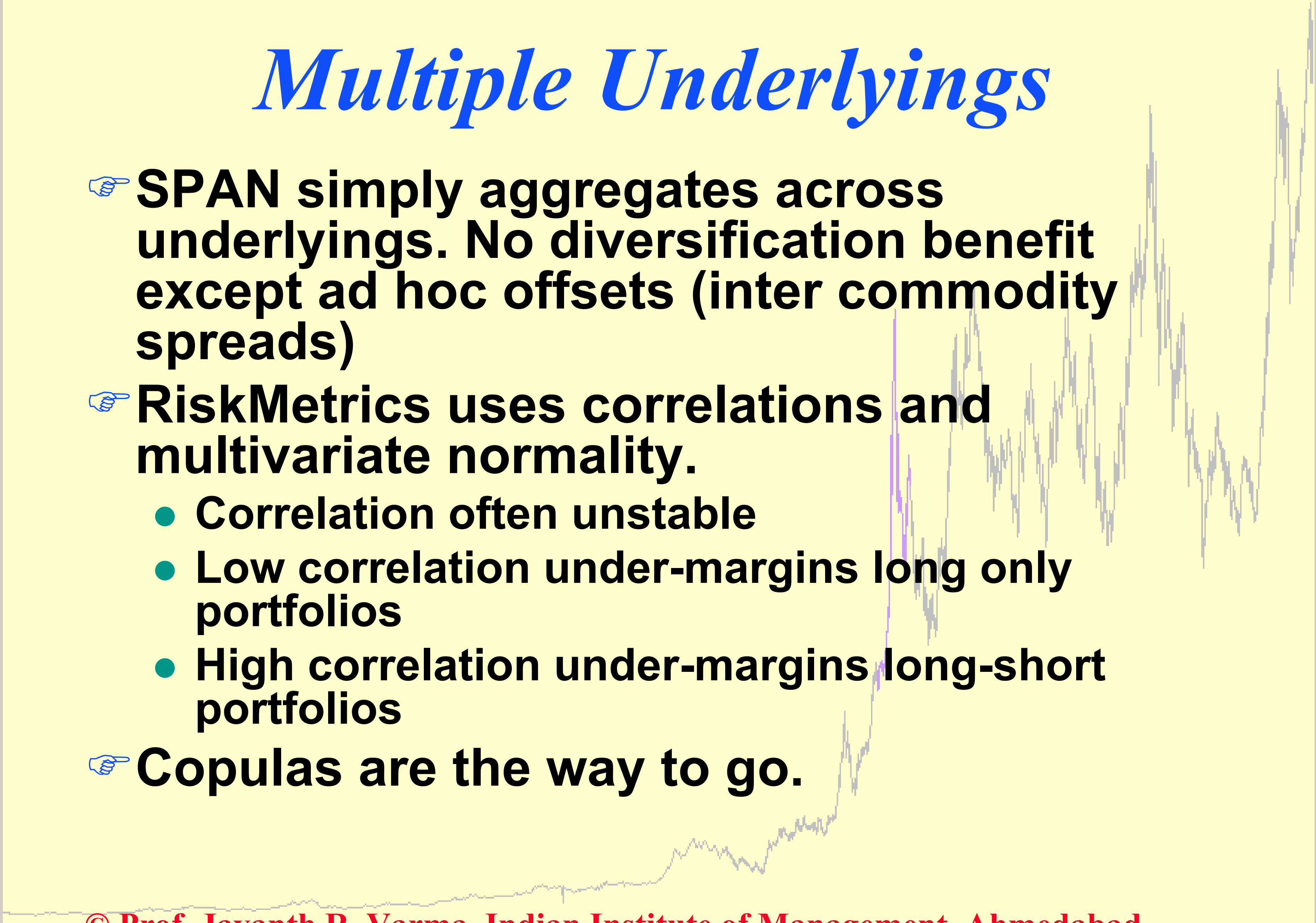
- SPAN is not portfolio VaR, it is more like sum of VaRs eg deep OTM call and put. It is a move towards ES.
- Delta-Gamma approximation can be used to compute ES by analytically integrating the polynomial over several sub intervals.
- In the tails, ES can be approximated using tail index: h/(h-1) times VaR. Use notional value or delta for aggregation. Indian ETD does this.
- All this entails only O(n) complexity.

Tail Index



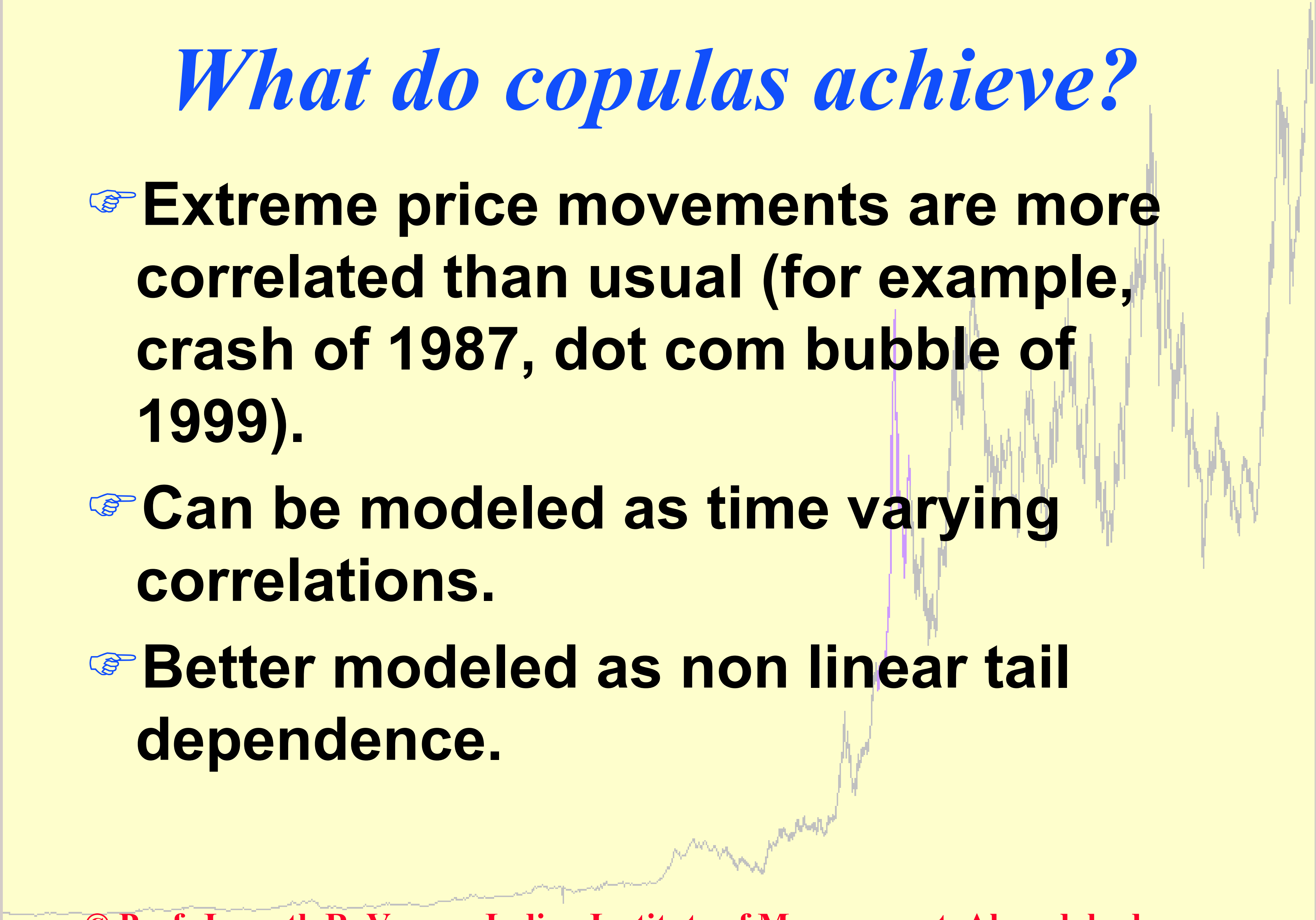
- Normal distribution has exponentially declining tails.
- Fat tails follow power law ~ x^{-h}
- Quasi Maximum Likelihood (QML):
  - Use least squares GARCH estimates
  - Estimate tail index from residuals
  - Consistent estimator + large sample size
- Risk Metrics is a GARCH variant

Multiple Underlyings

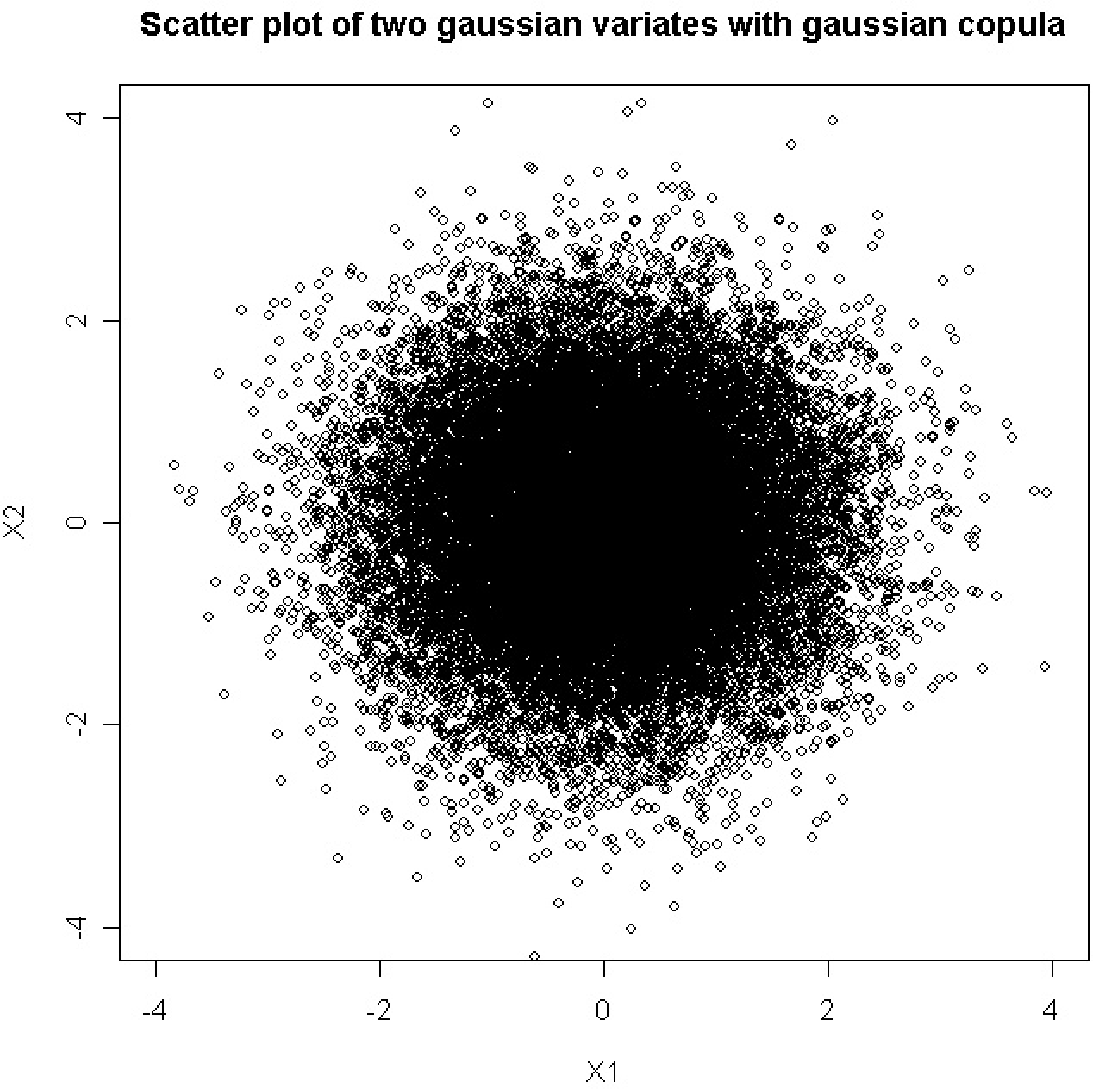


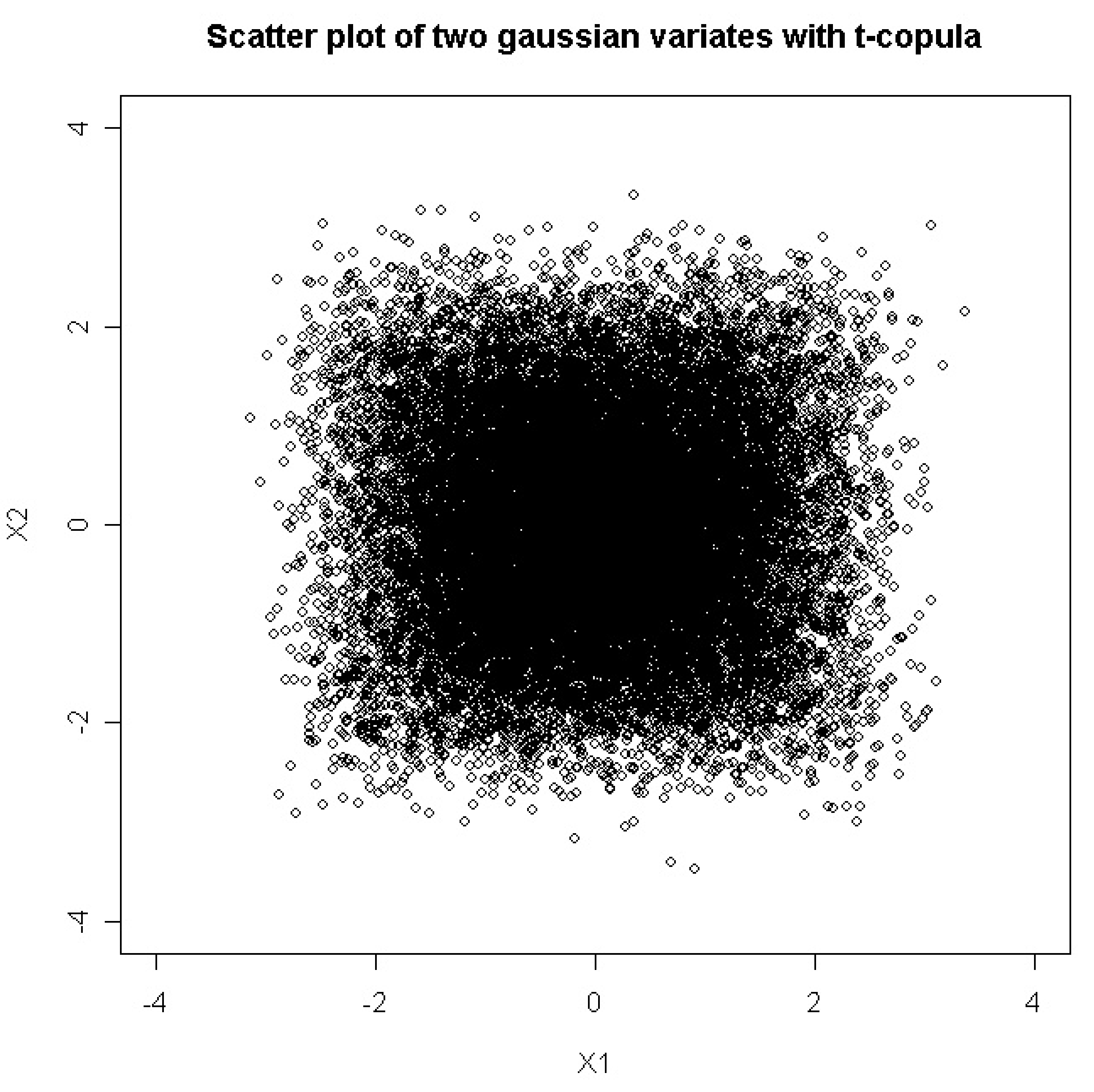
- SPAN simply aggregates across underlyings. No diversification benefit except ad hoc offsets (inter commodity spreads)
- RiskMetrics uses correlations and multivariate normality.
  - Correlation often unstable
  - Low correlation under-margins long only portfolios
  - High correlation under-margins long-short portfolios
- Copulas are the way to go.

What do copulas achieve?

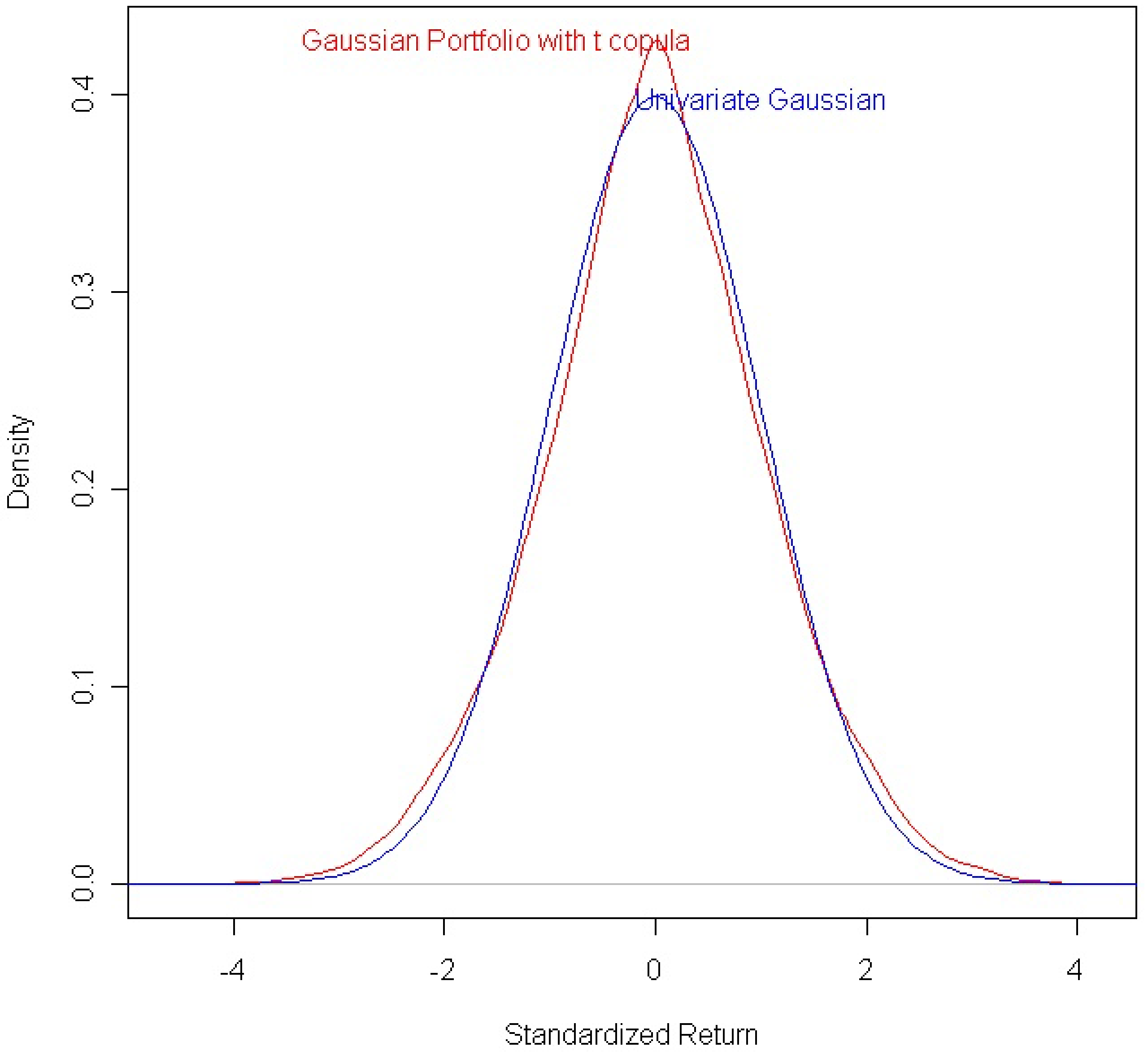


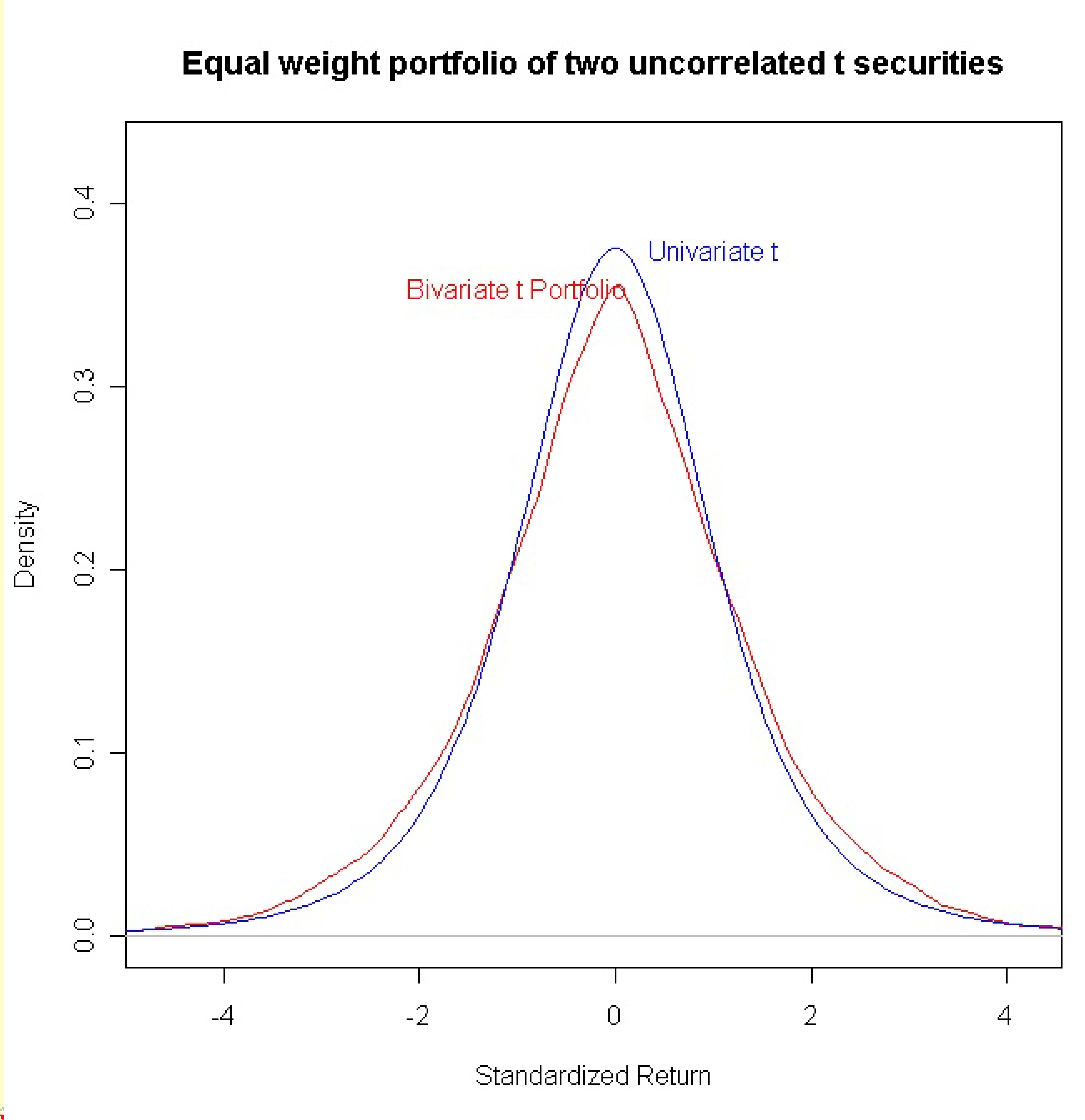
- **Extreme price movements are more correlated than usual (for example, crash of 1987, dot com bubble of 1999).**
- **Can be modeled as time varying correlations.**
- **Better modeled as non linear tail dependence.**


Scatter plot of two gaussian variates with gaussian copula

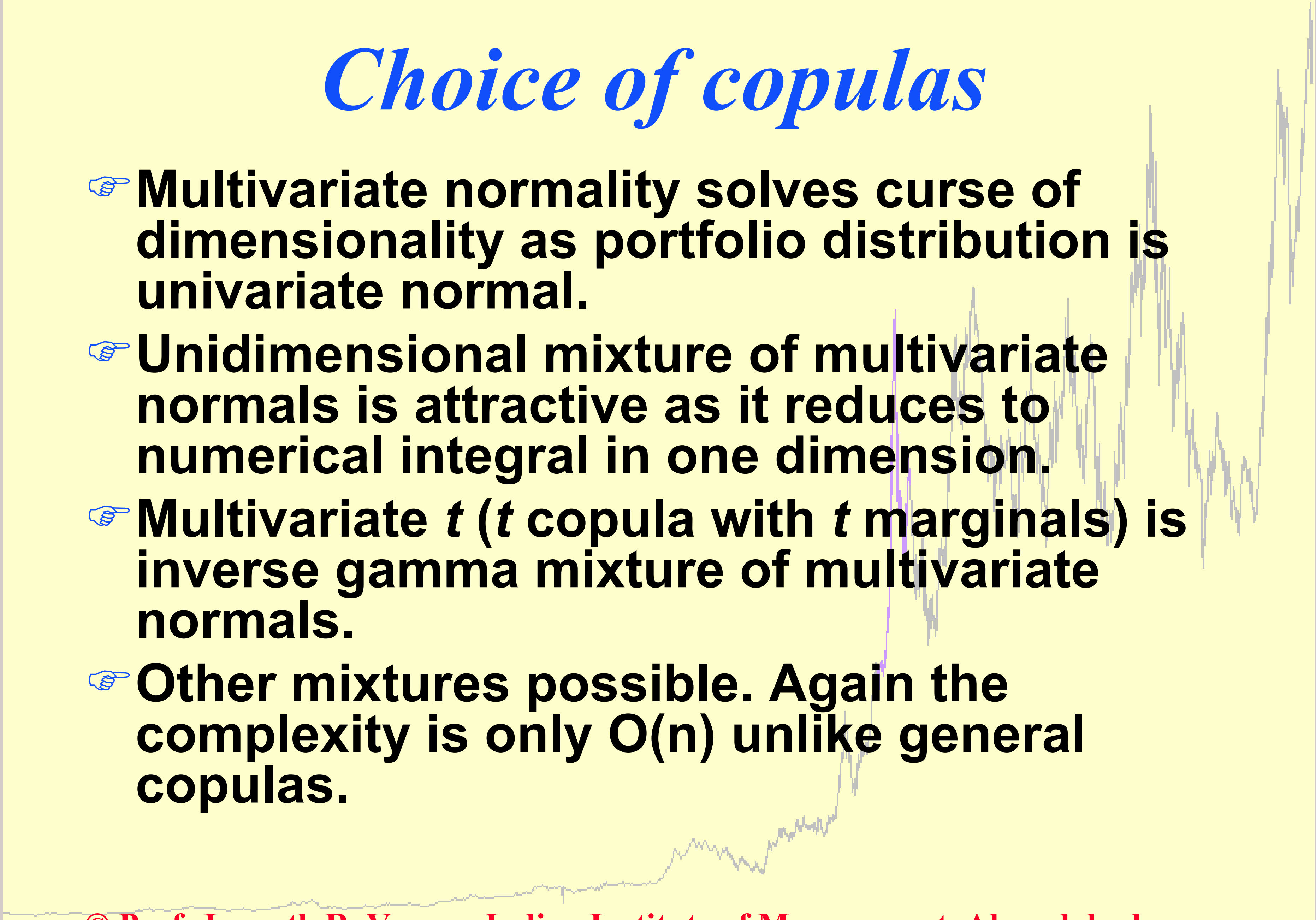

Scatter plot of two gaussian variates with t-copula
X2
4
2
0
-2
-4
-4
-2
0
2
4
X1

Equal weight portfolio of two uncorrelated guassian securities



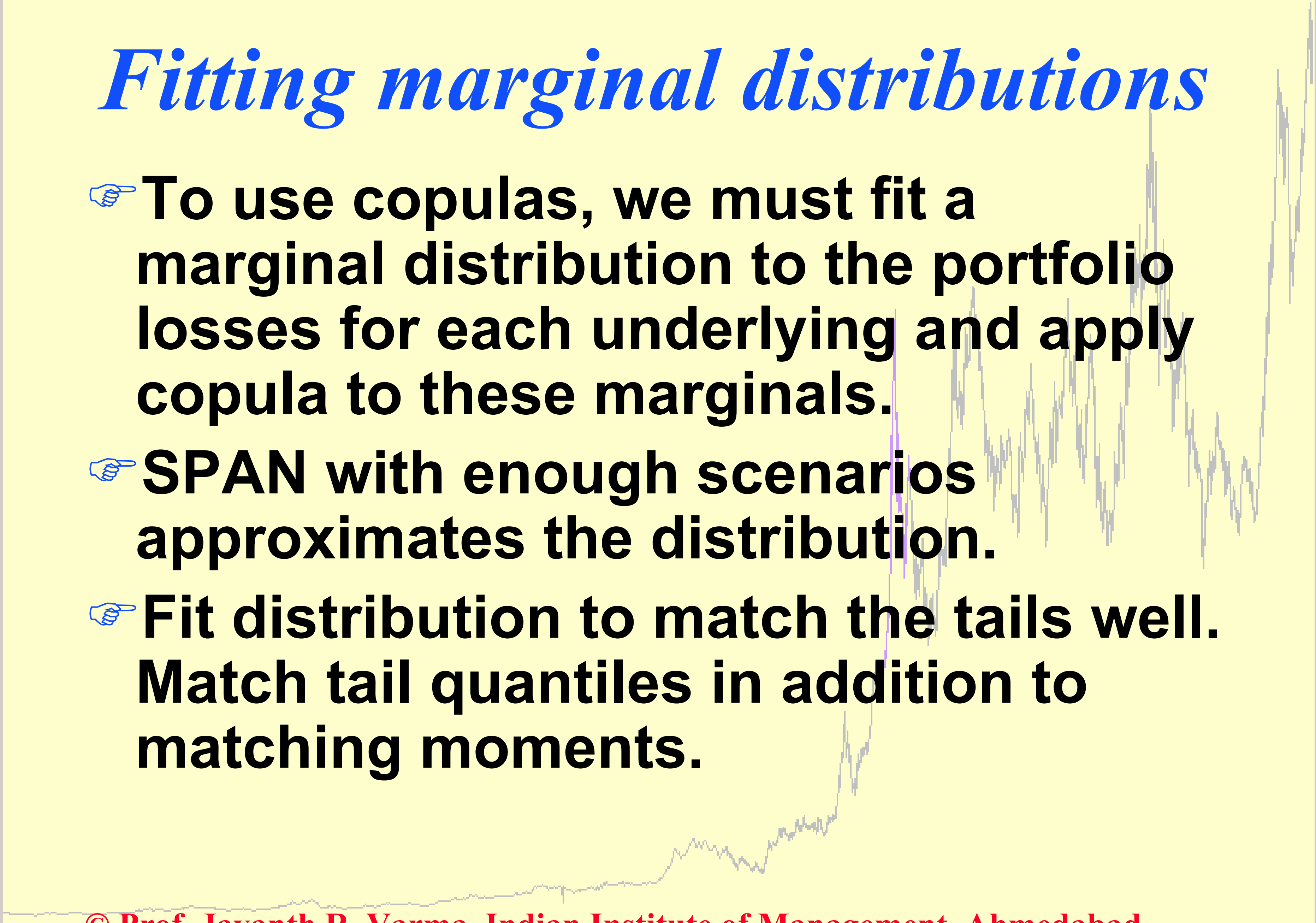

Equal weight portfolio of two uncorrelated t securities
Univariate t
Bivariate t Portfolio
Density
0.0
0.1
0.2
0.3
0.4
-4
-2
0
2
4
Standardized Return

Choice of copulas



- Multivariate normality solves curse of dimensionality as portfolio distribution is univariate normal.
- Unidimensional mixture of multivariate normals is attractive as it reduces to numerical integral in one dimension.
- Multivariate *t* (*t* copula with *t* marginals) is inverse gamma mixture of multivariate normals.
- Other mixtures possible. Again the complexity is only O(n) unlike general copulas.

Fitting marginal distributions

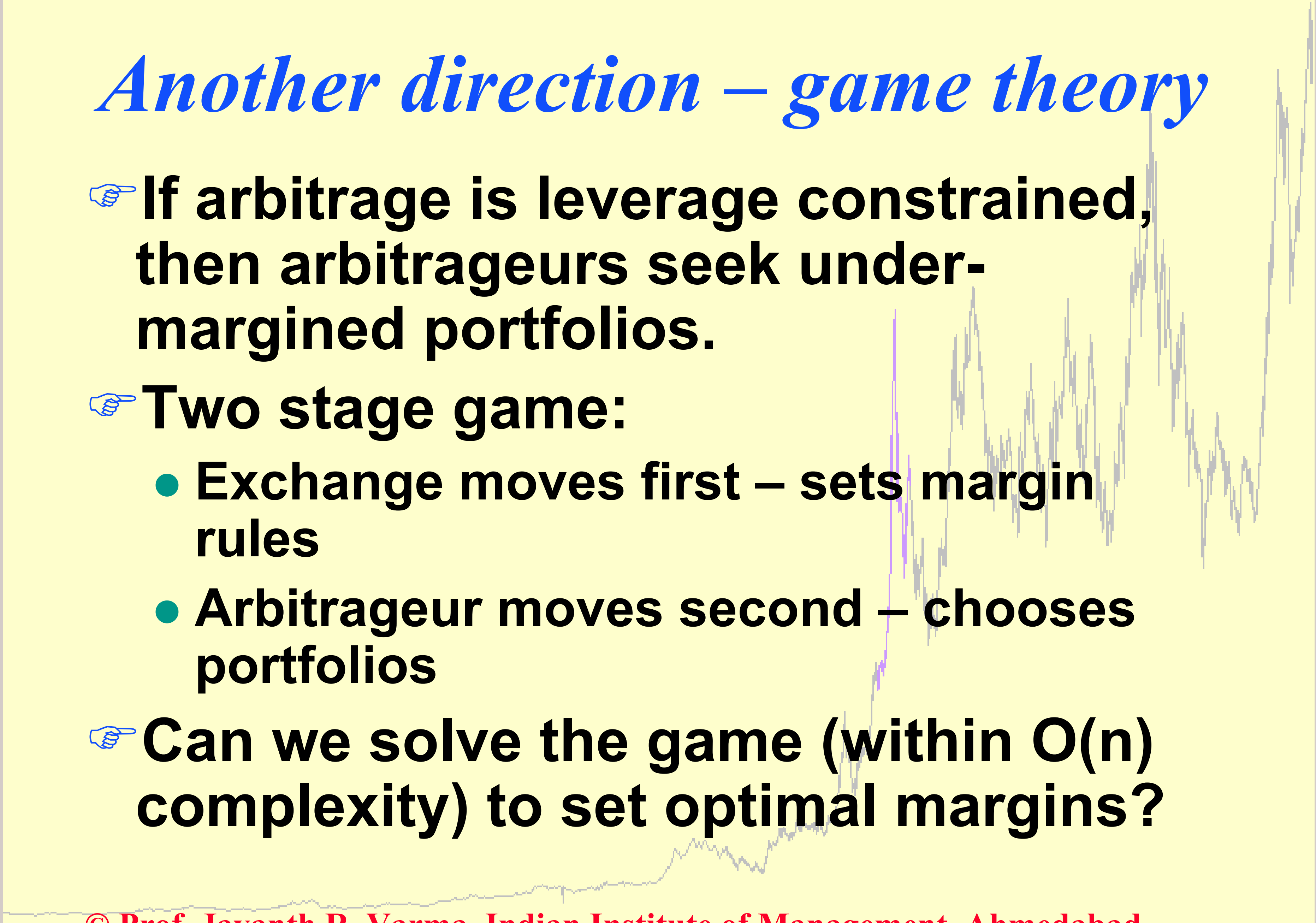


- To use copulas, we must fit a marginal distribution to the portfolio losses for each underlying and apply copula to these marginals.
- SPAN with enough scenarios approximates the distribution.
- Fit distribution to match the tails well. Match tail quantiles in addition to matching moments.

Directions for Research

- Statistical estimation and goodness of fit.
- Refinement of algorithms – accuracy and efficiency.
- Computational software (open source?)
- Advocacy.

Another direction – game theory



- If arbitrage is leverage constrained, then arbitrageurs seek under-margined portfolios.
- Two stage game:
  - Exchange moves first – sets margin rules
  - Arbitrageur moves second – chooses portfolios
- Can we solve the game (within O(n) complexity) to set optimal margins?

Game against nature

- Systemic risk:
  - Exchange is short options on each trader’s portfolio with strike equal to portfolio margin.
  - What price scenarios create worst loss to exchange (aggregated across all traders)?
  - Add these scenarios to margining system dynamically
- Three stage game:
  - Traders choose portfolios
  - Exchange decides on “special” margins or “special” margining scenarios
  - Nature (market?) reveals new prices
- Can we solve this game within O(n) complexity?